museum
Operational Directorate Natural Environment
OD Nature | OD Natuur | DO Nature

A multi-grid implicit scheme for time discretization in COHERENS: what and why

Overview

- What is a multi-grid implicit scheme
- Why apply a multi-grid implicit scheme
- What changes in COHERENS
- How does a multi-grid scheme work
- Features of the COHERENS multi-grid scheme
- The CSM test case with multi-grid scheme

What is a multi-grid implicit scheme

- Multi-grid scheme: algorithm for solving differential equations with a hierarchy of discretizations
- Explicit scheme: the state of the system at a later time computed from the system state at the current time
- Implicit scheme: solves an equation involving both the current state of the system and the later one

Why a multi-grid implicit scheme

- Explicit time stepping:
  - Slow
  - CFL condition (to keep the error bounded)

- Implicit time stepping:
  - Unconditionally stable
  - Large time steps can be handled

- Multi-grid scheme: accelerates the convergence of iterative methods by global correction from one time step to the next one by solving a coarse grid problem

museum
Operational Directorate Natural Environment
OD Nature | OD Natuur | DO Nature

What does change in COHERENS

- Switch: *iopt_hydro_impl*:
  - 0: explicit time discretization (mode splitting technique)
  - 1: implicit time discretization with multi-grid scheme

- ~~Mode splitting technique for time integration:~~
  - 2D depth integrated momentum and continuity equations solved with a time step small enough to satisfy the CFL condition
  - Results inserted in the 3D equations solved with a time step 10 to 20 times longer

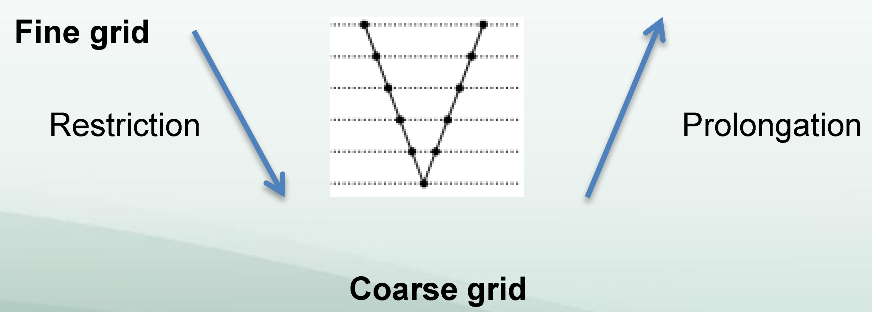
How does a multi-grid scheme work

- What you wish:
  - To solve T(u) = f
  - Solution: u
- Error: $e_m = u_m - u$
- Residual: $r_m = T_m(u_m) – f_m$

- Error:
  - high frequencies: removed in a few iterations
  - low frequencies: reduced very slowly
  - multi-grid idea: to change to a coarser grid where low frequencies act like higher frequencies

How does a multi-grid scheme work

- Three stages:
  - A restriction matrix: transfers vectors from the fine grid to the coarse grid
  - Iteration methods: on the coarser grid
  - A prolongation (interpolation) matrix: to return from the coarse to the fine grid:
  - Fine-coarse-fine loop: cycle



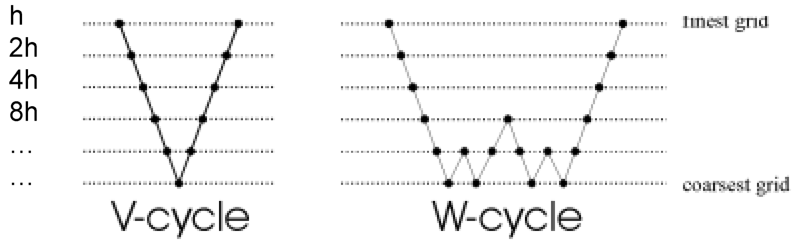
museum
Operational Directorate Natural Environment
OD Nature | OD Natuur | DO Nature

How does a multi-grid scheme work

- Algorithm:
    - Solve $T_m (u_m) = f_m$ on the fine grid
    - Compute residual on the fine grid: $r_m = T_m (u_m) – f_m$
    - Restrict the residual from the fine grid to the coarse grid
    - Cycle γ times (iteration) to minimize the residual on the coarse grid
    - Apply the coarse grid correction
    - Prolongate from the coarse grid to the fine grid

Features of the COHERENS multi-grid scheme

- Iterations until convergence: *iopt_mg_cycle*
  - 1: V-cycle
  - 2: W-cycle

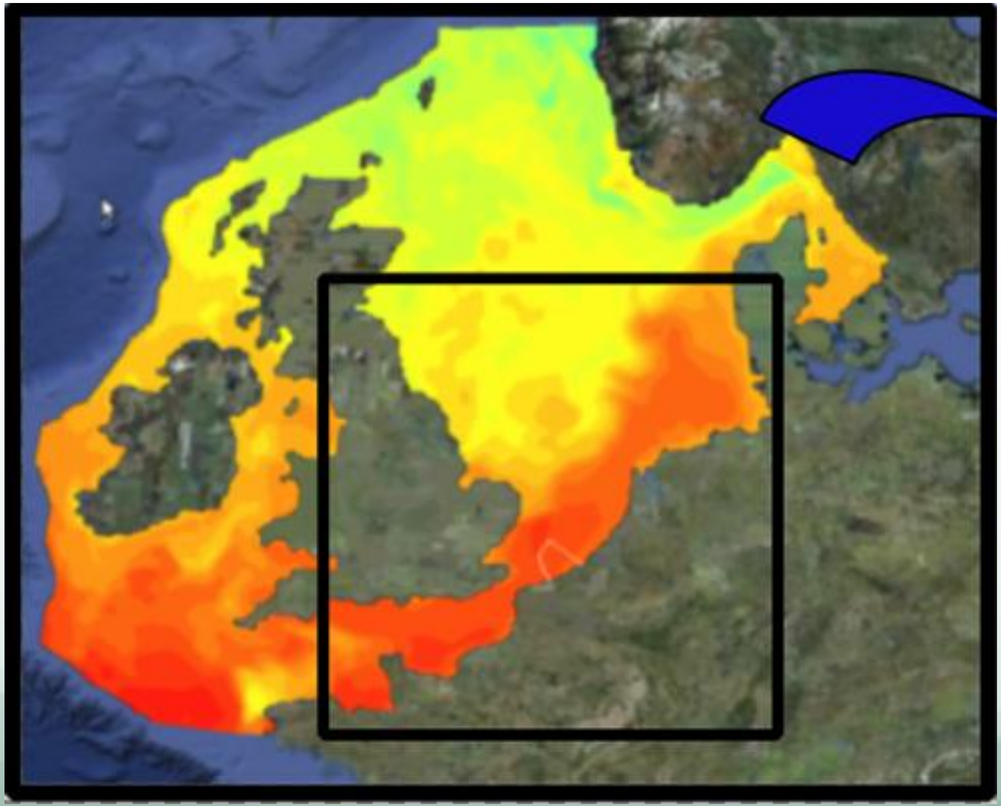


- Two iterative techniques: *iopt_mg_smoother*
  - 1: Jacobi
  - 2: Gauss-Seidel with red-black ordering: updates the even (red) points and then the odd (black) points

- Prolongation operators: *iopt_mg_prolong*
  - 1: injection
  - 2: bilinear interpolation

Application: the CSM test case

- North Sea Continental Shelf Model
- 2D simulations
- April 2006
- Tides:
  - 8 harmonic components
- Time step:
  - explicit: 20 sec
  - multigrid: 300 sec



museum
Operational Directorate Natural Environment
OD Nature | OD Natuur | DO Nature

The CSM test case: results

Scheme	Nb procs	Nb grid lev.	Cycle	Smoother	CPU time
Expl.	11	-	-	-	5042s.
Impl.	11	3	V	Jacobi	872s.
Impl.	11	3	V	G.-S.	889s.
Impl.	11	3	W	Jacobi	894s.
Impl.	11	3	W	G.-S.	1010s.